

Dynamics-dependent criticality in models with q absorbing statesAdam Lipowski^{1,2} and Michel Droz¹¹*Department of Physics, University of Genève, CH 1211 Genève 4, Switzerland*²*Department of Physics, Adam Mickiewicz University, 61-614 Poznań, Poland*

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We study a one-dimensional, nonequilibrium Potts-like model that has q symmetric absorbing states. For $q=2$, as expected, the model belongs to the parity-conserving universality class. For $q=3$ the critical behavior depends on the dynamics of the model. Under a certain dynamics it remains generically in the active phase, which is also the feature of some other models with three absorbing states. However, a modified dynamics induces a parity-conserving phase transition. Relations with branching-annihilating random walk models are discussed in order to explain such a behavior.

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I. INTRODUCTION

Recently, dynamical and nonequilibrium properties of many-body systems have been intensively studied. Of particular interest are nonequilibrium phase transitions that might appear in the stationary state of such systems [1]. It is believed that continuous phase transitions can be classified into relatively few universality classes. It is becoming evident, however, that such a classification is far more complicated in nonequilibrium systems than in equilibrium ones. In the equilibrium case, renormalization group and conformal-invariance theories gave a powerful description of this universal phenomena. In nonequilibrium, the situation is far less clear due to the lack of a general theory. Accordingly, a large body of works is based on numerical simulations.

Models with absorbing states constitute a particularly rich class. For these models there is a substantial evidence that continuous phase transitions can be classified into some universality classes. In particular, a large group of models falls into the so-called directed percolation (DP) universality class and it was conjectured that all models with a single absorbing state, positive one-component order parameter and short-range dynamics should generically belong to this universality class [2]. Another group consists of models having a double degenerate absorbing state or whose dynamics obey some conservation law; they belong to the parity-conserving (PC) universality class [3].

The behavior of models with a larger number of absorbing states was also addressed in the literature. For example, Bassler and Browne examined a model with three absorbing states and concluded that depending on some parameters their model might exhibit DP or PC criticality [4]. However, the control parameters in their model (adsorption rates) introduce asymmetry between absorbing states when the critical point is approached. Consequently, one or two species are effectively expelled from the system upon approaching the critical point hence the DP or PC criticality is an expected feature of this model. Similarly, certain asymmetries are responsible for the DP criticality in yet another model with multiple absorbing states that was studied by Janssen [5].

To examine the role of degeneracy, one has to study models where the symmetry between absorbing states is not bro-

ken at the level of dynamics. Good candidates for such a system are certain multispecies generalizations of the contact process model [6,7]. In the two-species case these models exhibit the expected PC criticality. In the three-species case Hinrichsen suggested [7] that such models will always remain in the active phase. This property, which should also be true for models with a larger number of absorbing states, follows from an approximate relationship with the q -species, parity-conserving branching-annihilating random walk models (q -BARW2, where the number at the end indicates the number of offsprings) [8]. Recently, Hinrichsen's conjecture was numerically confirmed for models with three and four absorbing states [9].

One can thus expect that for a given dimensionality, the number of (symmetric) absorbing states is the relevant parameter determining the critical behavior of a given model. However, for nonequilibrium systems some other details of the dynamics as, e.g., exclusion effects [11,13] or certain local symmetries [14], might affect critical behavior. The goal of the present paper is to provide yet another example of the dynamics-dependent criticality. In particular, we show that for a one-dimensional model with three absorbing states its critical behavior is PC-like (instead of the expected q -BARW2-like), when certain local constraints on the dynamics of a model are introduced. Let us emphasize that these constraints, which can be regarded as a local symmetry breaking, do not violate the symmetry between absorbing states [10]. We also suggest a mechanism that could explain such a behavior.

Our work shows that not only global properties of a given model, as the number of absorbing states, but also certain details of the dynamics are relevant to determine its critical behavior. Accordingly, classification of the critical properties of models with absorbing states is more complicated than originally thought.

The critical behavior of q -BARW2 models was recently found to be richer than originally expected. Indeed, in one dimension hard-core effects are known to change the off-critical exponents of the model [11,12]. Let us notice that these effects do not change the location of the critical point and of the on-critical exponents [13]. At the coarse-grained level, our model can be also regarded as a certain q -BARW2

model. In our case, however, dynamical details have more dramatic effect: they change both the location of the critical point and values of all critical exponents.

II. MODEL AND ITS MONTE CARLO SIMULATIONS

Before defining our dynamical model, let us recall some basic properties of the usual equilibrium Potts model. First we assign at each lattice site i a q -state variable $\sigma_i = 0, 1, \dots, q-1$. Next, we define the energy of this model through the Hamiltonian,

$$H = - \sum_{i,j} \delta_{\sigma_i \sigma_j}, \quad (1)$$

where summation is over pairs of i and j that are usually nearest neighbors and δ is the Kronecker δ function. This equilibrium model was studied using many different analytical and numerical methods and is a rich source of the information about phase transitions and critical phenomena [15].

To simulate numerically the equilibrium Potts model defined using the Hamiltonian (1) one introduces a stochastic Markov process with transition rates chosen in such a way that the asymptotic probability distribution is the Boltzmann distribution. One possibility of choosing such rates is the so-called Metropolis algorithm. In this method [16] one looks at the energy difference ΔE between the final and initial configuration and accept the move with probability $\min\{1, e^{-\Delta E/T}\}$, where T is temperature measured in units of the interaction constant of the Hamiltonian (1), which was set to unity. To obtain a final configuration one selects randomly a site and its state (one out of q in our case). In the above described algorithm for $T > 0$ there is always a positive probability of leaving any given configuration (even when the final configuration has a higher energy). Accordingly, such a model does not have absorbing states for $T > 0$.

A. A model

To transform the standard Metropolis dynamics into the dynamics with absorbing states we make the following modification.

Restriction A. When all neighbors of a given site are in the same state as this site, then this site cannot change its state (at least until one of its neighbors is changed).

In the following, this nonequilibrium model will be referred to as A model. Obviously, any of q ground states of model (1) is an absorbing state of A model and the dynamics does not favor any of the absorbing states. Since the dynamics of our models is obtained from a modification of the Metropolis algorithm of an equilibrium system, transition probabilities are parametrized by temperaturelike quantity T . Strictly speaking, for our model the ordinary (i.e., equilibrium) temperature cannot be defined. Nevertheless, we will refer to this quantity as temperature.

To study the properties of this A model we performed Monte Carlo simulations (MCS). A natural characteristic of models with absorbing states is the steady-state density of active sites ρ . A given site i is active when at least one of its neighbors is in a state different than i . Otherwise the site i is

called nonactive. In addition to the steady-state density we also looked at its time dependence $\rho(t)$. In the active phase $\rho(t)$ converges to the positive value while at criticality, $\rho(t)$ has a power-law decay $\rho \sim t^{-\delta}$. In the absorbing phase the density ρ decays either faster than a power of t or as a power of t but with a different exponent than the critical exponent δ . The unit of time is defined as a single (on average) update of each active site.

In addition, we used the so-called dynamic Monte Carlo method where one sets the system in the absorbing state, locally initiate activity, and then monitor some stochastic properties of surviving runs [17]. The most frequently used characteristics are the survival probability $P(t)$ that the activity survives at least until time t and the number of active sites $N(t)$ (averaged over all runs). At criticality these quantities are expected to have power-law decay: $P(t) \sim t^{-\delta'}$ and $N(t) \sim t^\eta$. (For some models $\delta = \delta'$, but exceptions are also known [1].)

Our simulations were made for various system sizes and we ensured that the system was large enough so that our results are size independent.

The simplest case is to consider a one-dimensional chain. However, for $q=2$ and for any temperature T , this model is trivially equivalent to the $T=0$ temperature Ising model with Metropolis dynamics. Indeed, in this case the allowed moves are only those, which do not increase energy and are always accepted. The same rule governs the dynamics of the $T=0$ Ising chain.

To overcome this geometrical ‘‘pathology’’ we consider our A model on a ladderlike lattice where two chains are connected by interchain bonds such that each site has three neighbors. Figure 1 illustrates the possibility of spreading of activity in the ladder geometry for $q=2$. The results of the simulations of such case are in agreement with the up-to-date knowledge concerning one-dimensional systems with multi-absorbing states. Thus we only briefly describe our results. For $q=2$ the model has two symmetric absorbing states (all $\sigma_i = 0$ or 1). Qualitatively, in this case the model resembles other models with two absorbing states [6,7], which are known to belong to PC universality class and our simulations confirm such a behavior.

First, the steady-state measurement of the density of active sites suggests a continuous phase transition between active and absorbing phases around $T=2.7$ (see Fig. 2). Measuring the time dependence of $\rho(t)$ we observed that at $T=2.7$ $\rho(t) \sim t^{-\delta}$ where δ is very close to the PC value 0.286. Moreover, in the low-temperature ($T < 2.7$) phase $\rho(t) \sim t^{-0.5}$, which is also a typical feature of PC models. Additional confirmation of PC criticality in this case is obtained using the dynamical Monte Carlo method that yields $\delta' = 0.29(3)$ and $\eta = -0.02(3)$.

A different behavior appears in the $q=3$ case. Although in Fig. 2 one can see a sudden change of the order parameter around $T=0.8$, there is no phase transition in this case. Examining the behavior of $\rho(t)$, we checked that even at low temperature ($T=0.5$ and 0.6) the system remains in the active phase. In addition simulations suggest that for $q=4$ and 5 A model behaves similarly to the $q=3$ case.

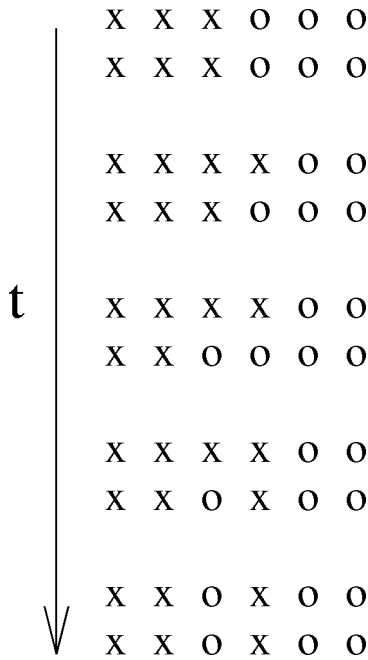


FIG. 1. At positive temperature a single domain can branch into additional domains. Successive (in time) configurations differ only by single-site flips. States 0 and 1 are represented by “x” and “o,” respectively. In the single-chain geometry and $q=2$ such a branching is forbidden and domain walls can only diffuse or annihilate. In terms of BARW models, where each domain wall represents a particle, the above sequence is equivalent to branching.

Actually, we expect that for $q \geq 3$ the model has a critical point but only at $T=0$. This point corresponds to the case of zero branching rate in a q -BARW2 model, which is known to be characterized by, e.g., $\beta=1$ (with the order parameter expressed in terms of the branching rate) [9].

As we already mentioned, the absence of the transition for $q \geq 3$ and $T > 0$ is an expected feature. However, as we will show below, additional restriction in the dynamical rules of our model induces a transition even for $q \geq 3$.

B. B model

This restriction can be formulated as follows.

Restriction B. A flip into a state different from any of its neighbors is prohibited. In other words, spontaneous creation of, for example, domains of type *A* between domains of type *B* and *C* is forbidden. Here, *A*, *B*, and *C* denote three (out of q) different states.

Let us notice that restriction *B* satisfies at the same time restriction *A* (but of course not vice versa). In the following we will refer to the model satisfying restriction *B* as *B* model. Let us also notice that restriction *B* does not break the symmetry and *B* model similarly to *A* model has q symmetric absorbing states. Results of the simulations of *B* model for $q=3$ are shown in Figs. 2–5 (for $q=2$ both dynamics *A* and *B* are equivalent). First, let us notice that the density ρ (Fig. 2) is only slightly larger for *B*-model than for *A*-model with $q=2$ (the difference is, however, larger than error bars). In

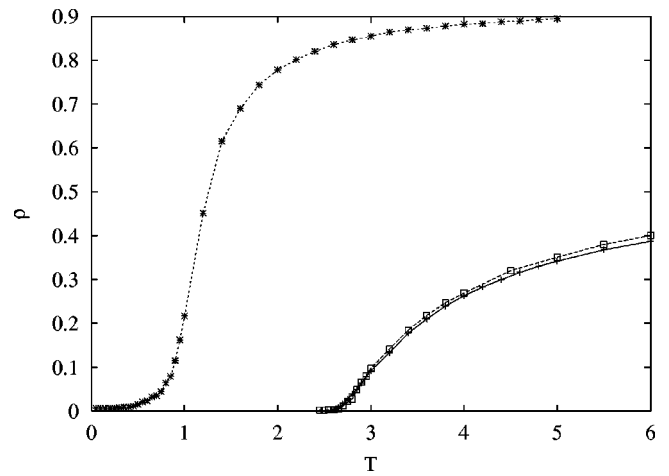


FIG. 2. The steady-state density ρ as a function of T for *A* model and $q=2$ (+), 3 (*) and for *B* model with $q=3$ (□). Simulations were made for the system size $L=30\,000$ and simulation time $t=10^5$ MCS. The low-temperature tail for $q=3$ does not diminish for longer simulations or larger system size.

addition, this difference diminishes upon approaching the transition point and within our numerical accuracy both transitions seem to take place at the same temperature. Later on we provide some arguments that could explain this apparent coincidence.

The behavior of $\rho(t)$ (Fig. 3) is typical for PC universality class. In the low-temperature phase ($T < 2.7$) we observe the power-law decay $\rho(t) \sim t^{-1/2}$ while at criticality ($T=2.7$) $\rho(t) \sim t^{-\delta}$ and we estimate $\delta=0.29(2)$. The dynamical Monte Carlo method confirms the PC criticality of this model [18]. Indeed, $P(t)$ scales with the exponent $\delta' = 0.29(2)$ (Fig. 4) and at criticality $N(t)$ remains virtually constant (Fig. 5), which is in agreement with the PC value $\eta=0.0$. In our opinion, the above results clearly indicate the PC criticality of *B* model. This behavior can be qualitatively

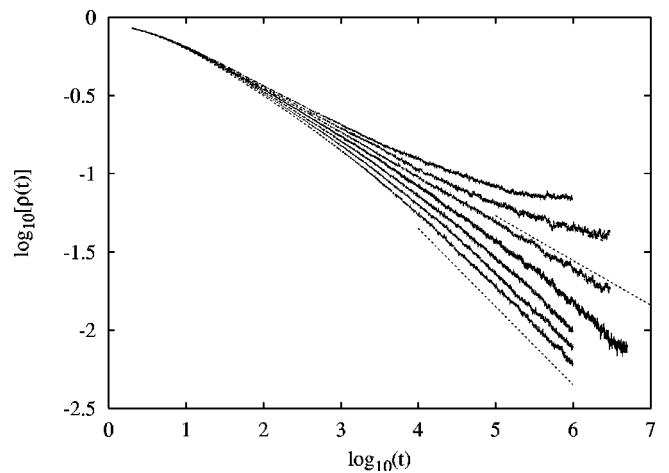


FIG. 3. The time dependence of the density $\rho(t)$ for *B* model with $q=3$ and (from top) $T=2.9, 2.8, 2.7$ (critical), 2.6, 2.5, 2.4, and 2.3 ($L=20\,000$). Each line is an average of 100 independent runs which starts from random initial configurations. Straight dotted lines have slopes corresponding to $\delta=0.286$ and 0.5.

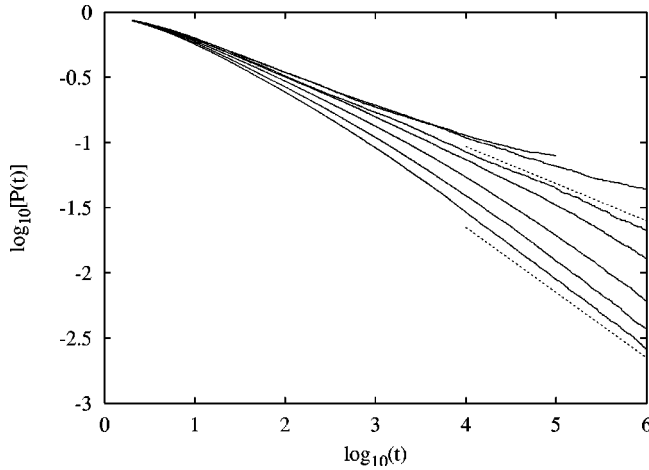
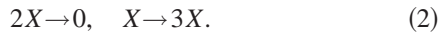


FIG. 4. The time dependence of the survival probability $P(t)$ for B model with $q=3$ and (from top) $T=2.9, 2.8, 2.7$ (critical), $2.6, 2.4, 2.2$, and 2.0 ($L=50\,000$). Each line is an average of about 10^6 independent runs. Straight dotted lines have slopes corresponding to $\delta=0.286$ and 0.5 , respectively.

explained by considering the relation between our models and certain multispecies parity-conserving q -BARW2 models.

The relation with such models is based on the observation that each domain wall can be at least approximately identified as a branching and annihilating random walker. Although precise mappings between absorbing states and BARW models are usually complicated, one can argue [7] that most relevant processes are only long-lived ones which considerably simplifies the resulting BARW model. In particular, Hinrichsen argued that dynamics of models with two absorbing states should be approximately described by the following BARW2 model [7]:



This model is known to belong to the PC universality class [19]. Since there are several types of domain walls, mappings for models with more than two absorbing states require several types of random walkers. Hooyberghs et al. argued that typically in the corresponding q -BARW2 models the following reactions should be included [9]:



The process (3) represents the formation of a domain of, e.g., type C between domains of type A and B . The process (4) is its reverse and corresponds to the disappearance of the domain C . The q -species BARW2 model (2)–(4) differs from the one studied by Cardy and Täuber [8]. However, the differences do not affect the critical behavior that is the same for both models. We expect that our A model for $q>2$ at the coarse-grained level is also described by q -BARW model with reactions (2)–(4).

Let us now notice that in our B model, the formation of the intruding domain C between domains A and B is forbid-

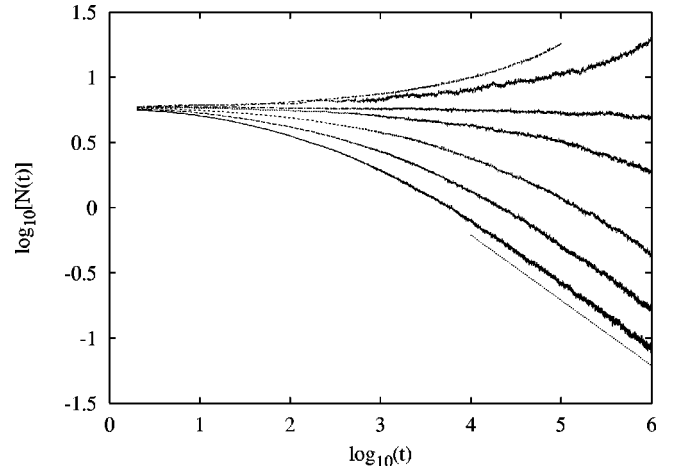


FIG. 5. The time dependence of the number of active sites $N(t)$ for B model with $q=3$ and (from top) $T=2.9, 2.8, 2.7$ (critical), $2.6, 2.4, 2.2$, and 2.0 ($L=50\,000$). Each line is an average of about 10^6 independent runs. The straight dotted line has a slope corresponding to $\delta=0.5$.

den, hence processes of the type (3) are forbidden. The q -species BARW2 model that corresponds to B model is thus described by reactions (2) and (4). The main point of our argument to explain the PC criticality of the B model is the following: The suppressing of the processes (3) implies separation of time scales of parity-conserving processes (2), which happen at the much shorter time scale than parity-nonconserving processes (4). As a result the q -BARW2 model spatially decomposes into single-species “clouds.” The dynamics within each “cloud” corresponds exactly to the dynamics of the $q=2$ B model (which is exactly the same as for A model), which at the coarse-grained level is given only by parity-conserving processes (2). Nonconserving processes (4) operate only when different clouds collide [20]. Upon approaching the transition point domains coarsen and the distances between clouds increase. As a result, non-conserving processes happen only on a very long time scale. (Snapshots of Monte Carlo simulations qualitatively confirm formation of such single-species clouds). On the other hand, the order parameter of the system, i.e., the number of particles is mainly determined by the parity-conserving processes (these processes determine the concentration of particles within clouds). As a result, the most relevant dynamics of the model is dominated by the parity-conserving processes that implies PC criticality. As we already mentioned, these processes correspond to the coarse-grained dynamics of the $q=2$ model, which thus explains why transition temperatures for $q=2$ and 3 are the same. It would be interesting to confirm these qualitative considerations with more sound theoretical arguments.

III. CONCLUSIONS

In conclusion, we have shown that the critical behavior of a model with q absorbing states is not only characterized by the number of absorbing states but that details of the dynam-

ics are also important. At the coarse-grained level our model is equivalent to a certain q -BARW2 model. It would be interesting to check whether suppression of processes (3) in q -BARW2 model leads to a similar change of the critical behavior.

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